Name ______________________ Class ___________ Date ________

A

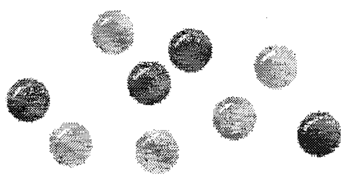
Explore It!

Probability of Independent and Dependent Events

Explore Probability With and Without Replacement

You can find the probability that a certain color of marble will be drawn from the group of marbles at the right. You can also determine the probability that a second marble of a given color will be drawn. The second probability depends on whether you replace the first marble or keep it separate from the group.

Activity 1



A bag contains 4 dark marbles and 5 light ones (as shown above).

1. You randomly draw one marble. What is the probability that it is light?

   ____

2. Assume you drew a light marble, then put it back in the bag. Again, you randomly draw one marble.

   How many light marbles are in the bag?________________________

   How many total marbles are in the bag?________________________

   What is the probability that that the marble you draw is light? Write a fraction for the probability.________________________________

3. Look at Step 2 again. Assume that instead of putting the light marble back in the bag, you set it aside. Again, you randomly draw one marble.

   How many light marbles are in the bag?________________________

   How many total marbles are in the bag? ________________________

   What is the probability that that the marble you draw is light? Write a fraction for the probability.________________________________

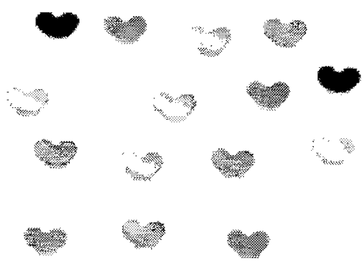
4. What is different about what you did in the second trial (Step 3) compared to what you did in the first trial (Step 2)?

____

____

Try This

A bag contains the following beans: 2 black, 5 white, 4 light gray, and 4 blue.



1. You draw 1 bean. What is the probability that it is black?

____

Assume that the first bean you drew was black. Here are three different questions about the second bean that you draw.

2. Without putting the black bean back in the bag, you draw a second bean. What is the probability that it is gray?

____

3. Without putting the black bean back in the bag, you draw a second bean. What is the probability that it is black?

____

4. You put the black bean back in the bag and draw a second bean. What is the probability that it is gray?

____

Draw Conclusions

5. Compare the probabilities in Exercises 2 and 4. Is the probability of drawing a gray bean on the second draw higher with or without replacement of the black bean?

6. Explain what is meant by drawing "with replacement" and "without replacement."

7. Why does the probability change when an object is drawn from a group without replacement?

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Hands-on LAB

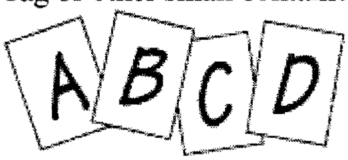
Experimental and Theoretical Probability

REMEMBER

- **The experimental probability of an event is the ratio of the number of times the event occurs to the total number of trials.**
- **The theoretical probability of an event is the ratio of the number of ways the event can occur to the total number of equally likely outcomes.**

Activity 1

1. Write the letters *A*, *B*, *C*, and *D* on four slips of paper. Fold the slips in half and place them in a bag or other small container.



2. Predict the number of times you expect to choose *A* when you repeat the experiment 12 times.
3. Without looking, choose a slip of paper, note the result, and replace the slip. Repeat this 12 times, mixing the slips between trials. Record your results in a table like the one shown.

Outcome	Number of Times Chosen
A	//
B	////
C	卌
D	/

4. How many times did you choose *A*? How does this number compare to your prediction?
5. What is the experimental probability of choosing *A*? What is the theoretical probability of choosing *A*?
6. Combine your results with those of your classmates. Find the experimental probability of choosing *A* based on the combined results.

Think and Discuss

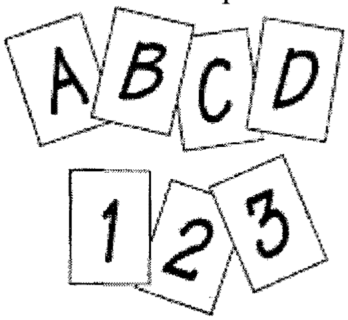
1. How is the experimental probability of choosing *A* based on the combined results different from the experimental probability of choosing *A* based on the results of your own experiment?
2. How many times would you expect to choose *A* if you repeat the experiment 500 times?

Try This

1. What is the theoretical probability of choosing *A* from five slips of paper with the letters *A, B, C, D,* and *E*? Predict the number of times you would expect to choose *A* if you repeat the experiment 500 times.
2. **Make a Conjecture** Based on your answers from problem 1, make a conjecture about experimental and theoretical probability if the number of trials is great.

Activity 2

1. Write the letters *A, B, C,* and *D* and the numbers 1, 2, and 3 on slips of paper. Fold the slips in half. Place the slips with the letters in one bag and the slips with the numbers in a different bag.



2. In this activity, you will be choosing one slip of paper from each bag without looking. What is the sample space for this experiment? Predict the number of times you expect to choose *A* and 1 (*A*-1) when you repeat the experiment 24 times.

Outcome	Number of Times Chosen
A-1	/
A-2	𝍸
A-3	//
B-1	/

3. Choose a slip of paper from each bag, note the results, and replace the slips. Repeat this 24 times, mixing the slips between trials. Record your results in a table like the one shown.
4. How many times did you choose *A*-1? How does this number compare to your prediction?
5. Combine your results with those of your classmates. Find the experimental probability of choosing *A*-1 based on the combined results.

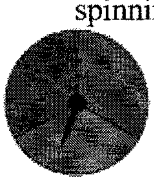
Think and Discuss

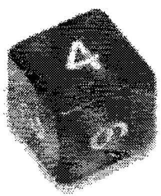
1. What do you think is the theoretical probability of choosing *A*-1? Why?
2. How many times would you expect to choose *A*-1 if you repeat the experiment 600 times?
3. Explain the difference between the experimental probability of an event and the theoretical probability of the event.

Try This

1. Suppose you toss a penny and a nickel at the same time.
   a. What is the sample space for this experiment?
   b. Predict the number of times you would expect both coins to land heads up if you repeat the experiment 100 times.
   c. Predict the number of times you would expect one coin to land heads up and one coin to land tails up if you repeat the experiment 1,000 times.

2. You spin the spinner at right and roll a number cube at the same time.
   a. What is the sample space for this experiment?
   b. Describe an experiment you could conduct to find the experimental probability of spinning green and rolling a 4 at the same time.





Name ____________________ Class __________ Date ________ C

Explore It!

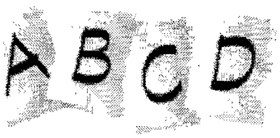
Making Predictions

Make a Prediction

A prediction based on the results of an experiment may be different than one based on theory. Suppose you toss a coin 10 times and get 7 heads. Based on your results, you should predict that you will get 70 heads out of 100 tosses, not 50 as theoretical probability would predict.

Activity 1

1. Write the letters *A*, *B*, *C*, and *D* on four slips of paper. Fold the slips in half and place them in a bag or other small container.



2. Predict the number of times you expect to choose *A* when you repeat the experiment 12 times.

   ____

3. Without looking, choose a slip of paper, note the result, and replace the slip. Repeat 12 times, mixing the slips between trials.

4. How many times did you choose *A*? ______________ How does this compare with your prediction?

   ____

5. Based on your results, about how many times would you expect to choose *A* if you repeated the experiment 500 times? Explain your reasoning.

   ____

   ____

Try This

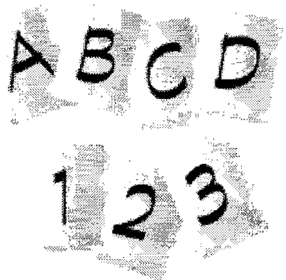
Liz wrote the letters *A*, *B*, *C*, *D*, and *E* on slips of paper. She drew and replaced letters 20 times.

1. About how many times would you expect her to choose *C*? ____________________
2. About how many times would you expect her to choose *C* in 100 trials?

   ____
3. Liz drew *C* seven times. Based on her results, about how many times would you expect her to choose *C* in 100 trials? ____________________

Activity 2

1. Write the letters A, B, C, and D and the numbers 1, 2, and 3 on slips of paper. Fold the slips in half. Place the slips with letters in one bag and the slips with numbers in a different bag.



2. In this activity, you will be choosing one slip of paper from each bag without looking. What is the sample space for this experiment?

   ____
3. Predict the number of times you would select *A*-1 in 24 trials.

   ____
4. Choose a slip of paper from each bag, note the results, and replace the slips. Repeat 24 times, mixing the slips between trials. Record your results in a table.
5. How many times did you choose *A*-1?

   ____

   How does this compare with your prediction?

   ____
6. Based on your results, about how many times would you expect to choose *A*-1 if you repeated the experiment 600 times? Explain your reasoning.

   ____

   ____

Try This

Tony tossed two coins at the same time.

4. What is the sample space for this experiment?

   ____

5. About how many times would you expect him to get two heads in 20 trials?__________________

6. In 20 trials, Tony got two heads 8 times. Based on those results, predict how many times he would get two heads in 200 trials.__________________________

Draw Conclusions

7. What do you think happens to the experimental and theoretical probability of an event when the number of trials is large?

   ____

   ____

D

Name ____________________ Class __________ Date ________

Explore It!

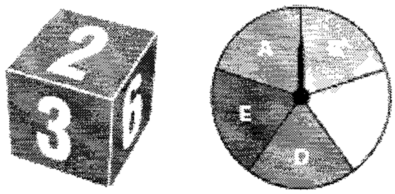
Sample Spaces

Find a Sample Space

The set of all possible outcomes of a probability experiment is called the *sample space*. The sample space may be quite small, as it is when you toss a coin (sample space: heads or tails).

Activity 1

A number cube is numbered from 1 to 6. A spinner has 5 equal sections lettered from A through E. You can use an organized list to find the sample space for an experiment.



1. Work with a partner. Decide on a logical system you can use to list all the possible outcomes when you roll the number cube and spin the spinner. Describe the method you will use.

____

____

____

2. Write the sample space. Use a number and a letter to write each possible outcome. For example, 3B means "Roll a 3, spin a B."

____

____

3. How many outcomes are there in the sample space? ____________________

Try This

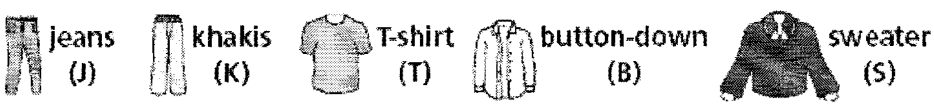
1. Suppose that instead of rolling a number cube in Activity 1, you tossed a coin.
   a. Write the sample space for tossing a coin (H = heads, T = tails) and spinning a spinner lettered from A through E.

   b. How many items are there in the sample space?____________________________

Sample spaces are easy to find in the real world. You may even have one in your own closet!

Activity 2

What would happen if you randomly chose your clothes in the morning? What pants-shirt combinations are possible outcomes?



1. Work with a partner again. Decide on a system you can use to list all the possible pants-shirt outfits if you have the clothes shown above. Describe the method you will use.

2. In the space below, write the sample space. Use two letters to write each possible outfit. For example, JS means "jeans and sweater."

3. What if you could wear each outfit with either brown shoes or black shoes? Without listing the sample space, explain how you could find the number of possible pants-shirt-shoe outfits. How many are there?

Try This

2. Ingrid is sending out thank you cards for birthday presents. She has pink, blue, and green cards, and white and yellow envelopes to send them in. She chooses a card and an envelope at random for each person. What is the sample space for possible combinations?

____

____

Draw Conclusions

3. In Try This 2, you found the sample space for three types of cards and two types of envelopes. What operation seems to relate three options and two options to the size of the sample space?

____

____

E

Name ____________________ Class __________ Date ________

Explore It!

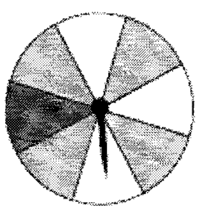
Theoretical Probability

Fair and Unfair Games

Have you ever felt that a game you were playing wasn't fair but you weren't sure why? You can use probability to decide whether a game is fair or unfair.

Activity 1

1. A spinner has 8 equal sections. Four sections are gray, three are white, and one is blue. Keri and Josh use the spinner to play a game. The rules are:
   - Players alternate turns.
   - Keri gets a point if she lands on gray or blue.
   - Josh gets a point if he lands on white or blue.
   - First player to earn 10 points wins.



2. Josh lost the first game and said the game wasn't fair. Keri said the 8 sections were equal, so the game was fair. What do you think of her argument? Does it prove that the game is fair?

____

____

____

3. If you were playing this game, would you take the gray and blue sections or the white and blue sections? Why?

____

____

Try This

1. Shane and Amanda play a number cube game. Shane gets a point if he rolls a number evenly divisible by 2. Amanda gets a point if she rolls a number evenly divisible by 3. Is the game fair? Explain.

____

____

____

Draw Conclusions

2. Change one rule in Keri and Josh's game to make the game fair.

____

Activity 2

1. Kara, Derik, and Heather alternate tossing two quarters. After each player's turn, one player receives a point, depending on the outcome of the tosses:
   - 2 heads: Kara gets 1 point
   - 2 tails: Derik gets 1 point
   - 1 head, 1 tail: Heather gets 1 point
2. Complete the table to show all the possible outcomes when you toss two coins. (Use H for heads and T for tails.)

First Coin	Second Coin
H	H

3. Use the results from the table. If you toss two coins:
   - On what percent of your tosses can you expect to get 2 heads? ____________
   - On what percent of your tosses can you expect to get 2 tails? ____________
   - On what percent of your tosses can you expect to get 1 head and 1 tail? ________

4. Is the game fair? Use your answers from Step 3 to explain why.

____

____

____

Try This

3. Change one rule in Kara, Derik, and Heather's game to make the game fair.

____

Draw Conclusions

4. Describe a fair game. Use the word *probability* in your description.

____

____

Name ______________________ Date ____________ Class Period ____F____

Hands-on LAB

Explore Permutations and Combinations

For some compound events, the order of the outcomes matters. For three outcomes *A, B,* and *C,* you need to know when *A-B-C* is different from *C-B-A* and when it is considered to be the same.

Activity 1

In how many different arrangements can Ellen, Susan, and Jeffrey sit in a row?

1. In this situation, the order of the students in the different arrangements is important. "Ellen, Susan, Jeffrey" is different from "Ellen, Jeffrey, Susan." An arrangement in which order is important is called a **permutation.**
2. Write each name on 6 index cards. You will have a total of 18 cards. Show all the different ways the cards can be arranged in a row.

Arrangement	1	2		4		6
First Seat	Ellen	Ellen	Susan	Susan	Jeffrey	Jeffrey
Second Seat	Susan	Jeffrey	Jeffrey	Ellen	Susan	Ellen
Third Seat	Jeffrey	Susan	Ellen	Jeffrey	Ellen	Susan

There are 6 different ways that these three people can sit in a row.
Another way to find the number of permutations is to multiply.

First Seat		Second Seat		Third Seat	
3 choices	×	2 choices	×	1 choice	= 6 permutations

Think and Discuss

1. Think of another situation in which the order in an arrangement is important. Explain.

Try This

1. Cindy, Laurie, Marty, and Joel are running for president, vice president, secretary, and treasurer of their class. Use multiplication to find the number of different ways the election can turn out.

Activity 2

Abe, Babe, Cora, and Dora are going to work on a project in groups of 2. How many different ways can they pair off?

1. In this situation, the order in the pairs is not important. "Abe, Cora" is the same as "Cora, Abe." When order is not important, the arrangements are called **combinations.**
2. Write each name on 3 index cards. You will have a total of 12 cards. Show all pairings.

Abe Babe	Babe Cora	Cora Dora
Abe Cora	Babe Dora	
Abe Dora		

There are 6 different possible pairs.

Think and Discuss

Tell whether each of the following is a permutation or a combination. Explain.

1. There are 20 horses in a race. Ribbons are given for first, second, and third place. How many possible ways can the ribbons be awarded?
2. There are 20 violin players trying out for the school band and 6 players will be chosen. How many different ways could students be selected for the band?
3. Connie has 10 different barrettes. She wears 2 each day. How many ways can she choose 2 barrettes each morning?
4. Yoko belongs to a book club, and she has just received 25 new books. How many possible ways are there for them to be placed on the shelf?

Try This

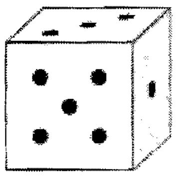
1. The video club is sponsoring a double feature. How many ways can club members choose 2 movies from a list of 6 possibilities?
2. Ms. Baker must pick a team of 3 students to send to the state mathematics competition. She has decided to choose 3 students from the 5 with the highest grades in her class. Ms. Baker can either send 3 equal representatives, or she can send a captain, an assistant captain, and a secretary. Which choice results in more possible teams? Explain. Find the number of teams possible for each choice.

9

Name ______________________ Date ____________ Class ________

Comparing Experimental and Theoretical Probability

KEY



A number cube contains the numbers 1 through 6 on each side. You can use a number cube to compute experimental and theoretical probabilities.

Activity

Roll a number cube 30 times. Record your results in the space below.

____

____

____

From your results, compute the experimental probability of getting a 1.

Count the number of ones in your results.

There were ________ ones in my results.

Thus, the probability is _________, which reduces to _________.

Compute the theoretical probability of getting a 1.

Find the total possible outcomes.

On a six-sided die, there are six possible outcomes.

Find the number of favorable outcomes.

Only getting a 1 is favorable, so there is one favorable outcome.

Write the fraction.

The fraction is $\frac{1}{6}$

Compare your results for the experimental and theoretical probability.

Are your results the same for both?

____

____

Think and Discuss

1. In the Activity, how many ones would you theoretically expect? Explain.

   ____

   ____

   ____

2. Why are the experimental and theoretical probabilities of an event often different?

   ____

   ____

   ____

   ____

Try This

1. Toss the number cube 30 times. Record your results below.

   ____

   ____

   ____

2. In the chart below, write the theoretical probability of each event. Then compute the experimental probability, based on your results.

Event: Probability of getting…	**Theoretical Probability**	**Experimental Probability**
a 6		
an even number		

a number less than 3		

3. Explain how your experimental probabilities differed from the theoretical.

Name ______________________ Class ________ Date ___________ H

Connect It!

Toss Up

When you toss a paper cup, it must land in one of three positions: face up, face down, or on its side.

1. Toss a paper cup 20 times. Record the outcomes.
2. What is the experimental probability that the cup lands face up? face down? on its side?

   ____

3. On any toss, is the cup more likely to land face up or face down? Why?

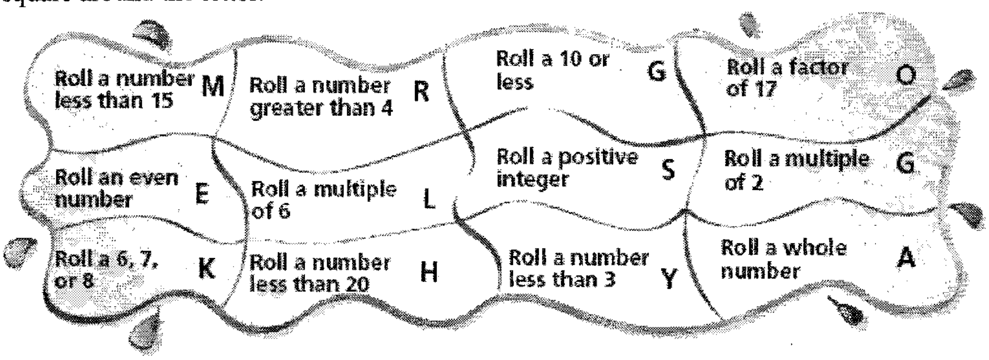
   ____

   ____

PUZZLER
Watch Your Tempera!

Tempera is a type of paint that has been used in art for centuries. The paint is made with a surprising ingredient. Solve the puzzle to find out what this ingredient is!

1. A spinner is divided into 20 equal spaces, numbered 1 through 20. Consider the following outcomes. If an outcome is as likely as not, draw a circle around the letter next to the outcome. If an outcome is unlikely, draw a square around the letter.



2. To discover the key ingredient in tempera paint, arrange the circled letters to find the first word of the answer. Arrange the letters around which you drew a square to find the second word of the answer.

   ____