A

Answer Key

Goal

Students consider several situations where two objects are chosen from a group, concluding that the probability that the second object will be chosen depends on whether or not the first object is replaced after it is chosen.

Teaching Tips

- To help students understand the difference between independent and dependent events, have students repeat the activity using 9 pieces of paper to represent the marbles.
- Point out that the probability of choosing a given color equals $\frac{\text{number of the given color}}{\text{total number of items}}$. If an item is removed from a group and not replaced, then the denominator of the fraction will be reduced by 1, thereby changing the probability of choosing the given color.

Building Conceptual Understanding

Ensure that students understand the difference between independent and dependent events, and that they can calculate the probability of each.

- Of 20 cards, 10 have the letter "A" on them and 10 have the letter "B." What is the probability of drawing "A" if you first draw an "A" and then replace it? What is the probability if you first draw an "A" but do not replace it? $\frac{1}{2}; \frac{9}{19}$
- In the Activity, you are asked to find the probability of drawing a light marble from a bag originally containing 4 dark marbles and 5 light ones. Why are the probabilities in Steps 2 and 3 different? **In Step 2, the light marble drawn in Step 1 is replaced, so there are 9 total marbles, 5 of which are light. In Step 3, the light marble drawn in Step 1 is not replaced, so there are only 8 total marbles, only 4 of which are light.**

Activity 1

1. $\frac{5}{9}$
2. 5; 9; $\frac{5}{9}$
3. 4; 8; $\frac{4}{8}$ or $\frac{1}{2}$
4. In the first trial, you put the light marble that you drew back in the bag. In the second trial, you did not put the marble back in the bag.

Try This

1. $\frac{2}{15}$
2. $\frac{4}{14} = \frac{2}{7}$
3. $\frac{1}{14}$
4. $\frac{4}{15}$

Draw Conclusions

5. Without replacement = $\frac{2}{7} \approx 0.29$; with replacement = $\frac{4}{15} \approx 0.27$; the probability without replacment is higher.
6. "With replacement" means that you replace an item that you removed from the set of possible outcomes, so that the number of outcomes remains unchanged. "Without replacement" means that you do not replace the item. The result is that the set of possible outcomes is reduced by one.
7. When an object is not replaced, the number of objects in the set of possible outcomes—the denominator and possibly the numerator of the fraction used to calculate the probability—is reduced by one.

B

Answer Key

Activity 1

2. Possible answer: 3

4. Check students' work.
5. Experimental probability: Check students' work; Theoretical probability: $\frac{1}{4}$
6. Check students' work.

Think and Discuss

1. The experimental probability based on the combined results is closer to the theoretical probability than the experimental probability based on the results of an individual's experiment.
2. Possible answer: 125

Try This

1. $\frac{1}{5}$, 0.2, 20%; Possible answer: 100
2. The greater the number of trials, the closer the experimental probability is to the theoretical probabilities.

Activity 2

2. Sample space: *A*-1, *A*-2, *A*-3, *B*-1, *B*-2, *B*-3, *C*-1, *C*-2, *C*-3, *D*-1, *D*-2, *D*-3; possible answer: 2

4. Check students' work.
5. Check students' work.

Think and Discuss

1. Possible answer: $\frac{1}{12}$; the experimental probability is close to $\frac{1}{12}$. Also, there are 12 equally likely outcomes in the sample space.
2. Possible answer: 50
3. The experimental probability is an approximation of the theoretical probability. In general, the more trials in an experiment, the more accurate the approximation.

Try This

1. a. HH, HT, TH, TT
   b. Possible answer: 25
   c. Possible answer: 500

2. a. red-1, red-2, red-3, red-4, red-5, red-6, blue-1, blue-2, blue-3, blue-4, blue-5, blue-6, green-1, green-2, green-3, green-4, green-5, green-6
   b. Possible answer:Spin the spinner and roll the number cube 100 times. Keep track of the number of times the spinner lands on green and you roll a 4. Divide this number by the number of trials (100).

Answer Key

Materials

Each student or group needs:

- 4 small slips of paper
- a bag or other small container.

Goal

Students conduct two probability experiments, comparing their results with the expected theoretical results and seeing that the two, and predictions based on the two, may be quite different.

Teaching Tips

- Some students may balk at the idea that the probability of tossing heads can be anything other than 50%-that tossing 9 heads in 10 trials, for example, means that the experimental probability of tossing heads is 90%. Stress that experimental probability is based on results of isolated experiments and can change in further experiements.

Building Conceptual Understanding

Ensure that students understand the difference between experimental and theoretical probability, and that they can make predictions based on the results of simple probability experiments.

- If you rolled a number cube 60 times, how many times would you expect to roll a 5? Explain. **10; There are 6 possible outcomes (1, 2, 3, 4, 5, 6), so you would expect any one of them on one-sixth of your rolls; $\frac{1}{6} \times 60 = 10$.**
- Kerry rolled a number cube 12 times and got 5 three times. Based on those results, how many times would you expect her to roll a 5 in 60 rolls? Explain. **25; She rolled 5 on threetwelfths or one-fourth of her rolls; $\frac{1}{4} \times 60 = 15$.**

Activity 1

2. Possible answer: 3
3. Check students' work.
4. Check students' work.
5. Check students' work. Answers should be based on the experimental probability from the activity.

Try This

1. about 4
2. about 20
3. 35

Activity 2

2. *A*-1, *A*-2, *A*-3, *B*-1, *B*-2, *B*-3, *C*-1, *C*-2, *C*-3, *D*-1, *D*-2, *D*-3

3. 2

5. Check students' work.

   Check students' work.

6. Check student's work. Answers should be based on the experimental probability from the activity.

Try This

4. HH, HT, TH, TT
5. 5
6. 80

Draw Conclusions

7. As the number of trials increases, the experimental probability will become closer to the theoretical probability.

D

Answer Key

Goal

Applying the strategy Make an Organized List, students count the possible outcomes of two probability experiments, using logical thinking to prepare themselves to understand and use formal counting methods.

Teaching Tips

- For each activity, emphasize the use of an organized counting system. Explain that listing possible outcomes randomly will take longer and will likely miss one or more outcomes. For Activity 2, for example, suggest that students focus on one top at a time with each of the different pants.
- Connect this topic back to probability by explaining that finding the probability of an outcome requires that you know how many different ways an event can occur.

Activity 1

1. Possible answer: Start with 1. List 1 with each of the 5 possible letters. Repeat with 2, 3, 4, 5, and 6 in succession.
2. 1A, 1B, 1C, 1D, 1E, 2A, 2B, 2C, 2D, 2E, 3A, 3B, 3C, 3D, 3E, 4A, 4B, 4C, 4D,4E, 5A, 5B, 5C, 5D, 5E, 6A, 6B, 6C, 6D, 6E
3. 30

Try This

1. a. HA, HB, HC, HD, HE, TA, TB, TC, TD, TE
   b. 10

Building Conceptual Understanding

Ensure that students can list all the possible outcomes of simple probability experiments using informal methods such as making an organized list or using logical thinking.

- Describe an organized method you could use to list all the possible outcomes if your toss a coin (H, T) and roll a number cube (1, 2, 3, 4, 5, 6). How many possible outcomes are there? **Possible answer: Start with H. Pair it with each possible number cube outcome (H1, H2, H3, H4, H5, H6). Repeat, pairing T with each possible number cube outcome (T1, T2, T3, T4, T5, T6); 12.**
- **Close** Adam, Brynn, and Cody (A, B, C) are running for class president. Dawn, Errol, and Fritz (D, E, F) are running for vice-president. List all the possible president/ vice-president outcomes. How many are there? **AD, AE, AF, BD, BE, BF, CD, CE, CF; 9**

Activity 2

1. Possible answer: Start with pants. List J with each of the 3 possible shirts. Repeat with K.
2. JT, JB, JS, KT, KB, KS
3. There are 12 pants-shirt outfits now. When I add shoes, there will be 6 outfits matched with black shoes and 6 outfits matched with brown shoes,for a total of 12 possible outfits.

Try This

2. PY, PW, GY, GW, BY, BW

Draw Conclusions

3. Possible answer: There are 6 options in the sample space, and you can also get that answer by multiplying 3 and 2.

E

Answer Key

Goal

By scrutinizing the rules of two games for fairness, students gain practice in real-world applications of probability, and a practical understanding of the meaning of a fair experiment.

Teaching Tips

- You may wish to have students work on Activity 1 in pairs. Pairs can play mock games to get a sense of how the rules work, with one player playing Keri and the other Josh.
- For Activity 2, group the students into groups of three. The three players can use two coins to actually play the game. They should toss a sufficient number of times to allow the advantage of "1 head, 1 tail" to become apparent.

Building Conceptual Understanding

Ensure that students understand the meaning of a "fair" game and an "unfair" game, and can analyze simple games of chance to decide whether or not they are fair.

- A spinner has 9 equal sections numbered 1 to 9. Player 1 receives a point if the arrow lands on an even number. Player 2 receives a point if it lands on an odd number. Is the game fair? Why or why not? **It is not fair. Player 2 has 5 ways of winning (1, 3, 5, 7, 9) but Player 2 has only 4 ways of winning (2, 4, 6, 8).**
- Describe a game involving a number cube that you could play with a friend and that would not be fair. Explain why it wouldn't be fair. **Possible answer: I score a point if I roll a number greater than 3. You score a point if you roll a number less than 3. I have more chances of winning (4, 5, 6) than you do (1, 2).**
- Change one rule in the game you described above to make the game fair. **Possible answer: I score a point if I roll a number greater than 4. You score a point if you roll a number less than 3.**

Activity 1

2. It's true that the sections are equal, but 5 of them are worth a point to Keri, while only 4 are worth a point to Josh. Keri's sections cover $\frac{5}{8}$ of the spinner, Josh's $\frac{4}{8}$.
3. Gray and blue; because there are 5 gray or blue sections but only 4 white or blue sections, you have a much better chance of winning with gray or blue.

Try This

1. Shane gets a point by rolling 2, 4, or 6. Amanda gets a point by rolling 3 or 6. Since Shane has a better chance of scoring points, and winning, than Amanda, the game isn't fair.

Draw Conclusions

2. Possible answer: Keri gets a point if she lands on gray.

Activity 2

2. H, T

   T, H

   T, T

3. • 25 %
   • 25 %
   • 50 %
4. No; Kara and Derik can expect to get a point on just 25% of their tosses. Heather can expect to get a point on 50% of her tosses. Because Heather has a better chance of scoring points than the others, the game isn't fair.

Try This

3. Possible answer: 1 head, 1 tail: Heather gets $\frac{1}{2}$ point

Draw Conclusions

4. Possible answer: A game is fair if each player's probability of winning is equal to every other player's probability of winning.

F

Answer key

Think and Discuss

Possible answer: The order is important when conducting a systematic sample in which every third person will be surveyed. Even if the first three people through the door are the same in each case, the order in which they enter affects the survey results. The order is not important in a situation in which the first three people into a store win a prize. All three people will win, regardless of who was first, second, or third to enter.

Try This

1. 24

Activity 2

Think and Discuss

1. Permutation; order is important.
2. Combination; order is not important.
3. Combination; order is not important.
4. Permutation; order is important.

Try This

1. 15
2. Choosing three equal team members is a combination problem and results in 10 possible teams; choosing a captain, an assistant captain, and a secretary is a permutation problem and results in 60 possible teams. The permutation problem has more possible teams because the order in which team members are chosen is important.

Answer Key

Activity

Answers will vary based on experiment results.; Possible answer: 1, 3, 2, 5, 1, 6, 4, 2, 3, 1, 4, 5, 6, 6, 1, 2, 5, 4, 5, 1, 3, 3, 4, 1, 6, 5, 6, 2, 3, 2; Possible answer: 6; $\frac{6}{30}$; $\frac{1}{5}$;

Possible answer: The results are different, but the experimental is close to the theoretical as expected.

Think and Discuss

1. Possible answer: You would expect 5 ones. Then the experimental probability would be $\frac{5}{30}$, or $\frac{1}{6}$, which is the same as the theoretical probability.
2. Possible answer: The theoretical probability tells what is expected on one trial. But since every outcome has some chance on every trial, it is unlikely that the results will exactly match what is expected over a number of trials.

Try This

1. Answers will vary based on experiment results. Answers below relating to experimental probability will use these numbers: 1, 3, 4, 2, 4, 5, 6, 6, 3, 2, 5, 1, 5, 3, 5, 4, 1, 6, 5, 6, 2, 4, 1, 4, 3, 6, 5, 6, 1, 3
2.

Event: Probability of getting...	Theoretical Probability	Experimental Probability
a 6	$\frac{1}{6}$	$\frac{1}{5}$
an even number	$\frac{1}{2}$	$\frac{7}{15}$
a number less than 3	$\frac{1}{3}$	$\frac{4}{15}$

3. Possible answer: The probability of getting a 6 was higher than the theoretical. The probability of getting an even number was slightly lower than the theoretical. The probability of getting a number less than 3 is slightly less than the theoretical.

4

Answer Key

Toss Up

Students conduct a simple experiment that has three outcomes. They record their results and then use experimental probability, as well as proportions, to analyze the experiment and make predictions. **Materials** — paper cup

PUZZLER

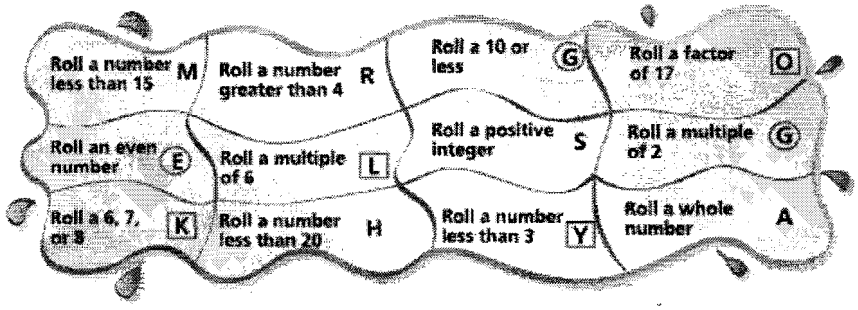
The puzzler requires students to determine the likelihood of a variety of events.

Toss Up

2. Possible answer: $\frac{1}{20}$; $\frac{1}{10}$; $\frac{17}{20}$
3. Possible answer: Face down because the experimental probability of landing face down is greater than the probability of landing face up.

PUZZLER
Watch Your Tempera!

1.



2. EGG YOLK