

Pre-Algebra GCD and LCM

Least Common Multiple (LCM)

Example A: Find the LCM of 12 and 18.

Step 1: Calculate the prime factors of each number and enter them into a small table:

$$\begin{array}{r}
 12 = 2 \times 2 \times 3 \\
 18 = 2 \times 3 \times 3 \\
 \hline
 \text{LCM} = \boxed{2 \times 2 \times 3 \times 3}
 \end{array}$$

So the $\text{LCM} = 2 \times 2 \times 3 \times 3 = 36$

Step 2: Line up the prime factors so that those common to all of the numbers are in the same column.

Step 3: Bring one factor from every column below the line.

Step 4: Multiply all of the numbers below the line to obtain the LCM.

Example B: Find the LCM of 6, 8 and 18.

$$\begin{array}{r}
 6 = 2 \times 3 \\
 8 = 2 \times 2 \times 2 \\
 18 = 2 \times 3 \times 3 \\
 \hline
 \text{LCM} = \boxed{2 \times 2 \times 2 \times 3 \times 3}
 \end{array}$$

So, the $\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 = 72$.

Lowest Common Denominator (LCD)

When fractions with different denominators are to be added or subtracted, it is necessary to find the Lowest Common Denominator. The LCD is essentially the Least Common Multiple of the denominators in question. Consider this problem:

Example: Calculate: $\frac{5}{12} - \frac{7}{18}$. In Example A, the LCM of 12 and 18 was calculated to be 36. To determine which **fractional name for 1** must be multiplied by each fraction to obtain a common denominator, we look for the missing numbers in each row. From Example A above:

$$\begin{array}{r}
 12 = 2 \times 2 \times 3 \quad \square \quad \text{missing a "3"} \\
 18 = 2 \times \square \times 3 \times 3 \\
 \hline
 \text{LCM} = 2 \times 2 \times 3 \times 3 = 36 \\
 \quad \quad \quad \text{missing a "2"}
 \end{array}$$

The missing number in the row for 12 is 3. Therefore, we use $\frac{3}{3}$ as a multiplier for $\frac{5}{12}$. Similarly, we use $\frac{2}{2}$ as a multiplier for $\frac{7}{18}$.

$$\text{Result: } \left(\frac{3}{3}\right) \left(\frac{5}{12}\right) - \left(\frac{7}{18}\right) \left(\frac{2}{2}\right) = \frac{15}{36} - \frac{14}{36} = \frac{1}{36}$$