

Pre-Algebra GCD and LCM

Simple methods for finding the Greatest Common Divisor (GCD) and the Least Common Multiple (LCM) are related, as shown below. Both involve developing a table of prime factors for the numbers in question. The methods are best illustrated by example.

Greatest Common Divisor (GCD)

Example A: Find the GCD of 180 and 105.

Step 1: Calculate the prime factors of each number and enter them into a small table:

$$\begin{array}{r}
 180 = 2 \times 2 \times 3 \times 3 \times 5 \\
 105 = \underline{\hspace{1.5cm} 3 \hspace{1.5cm} \times 5 \times 7} \\
 \text{GCD} = \hspace{1.5cm} 3 \hspace{1.5cm} \times 5
 \end{array}$$

So, $\text{GCD} = 3 \times 5 = 15$.

Step 2: Line up the prime factors so that those common to all of the numbers are in the same column.

Step 3: Bring any factors that show up for every number (i.e., that fill the column) below the line.

Step 4: Multiply all of the numbers below the line to obtain the GCD.

Example B: Find the GCD of 140, 210 and 462.

$$\begin{array}{r}
 140 = 2 \times 2 \times 5 \times 7 \\
 210 = 2 \times 3 \times 5 \times 7 \\
 462 = \underline{\hspace{1.5cm} 2 \hspace{1.5cm} \times 3 \hspace{1.5cm} \times 7 \times 11} \\
 \text{GCD} = 2 \hspace{1.5cm} \times 7
 \end{array}$$

So, $\text{GCD} = 2 \times 7 = 14$.

Example C: Find the GCD of 32 and 27.

$$\begin{array}{r}
 32 = 2 \times 2 \times 2 \times 2 \times 2 \\
 27 = \underline{\hspace{1.5cm} 3 \times 3 \times 3} \\
 \text{GCD} = (\text{there are no common factors})
 \end{array}$$

So, $\text{GCD} = 1$.

Calculating a Number's Prime Factors

In order to calculate the prime factors of a number, simply begin dividing it by primes, starting with 2 and working higher until all factors are primes.

Examples: Find the prime factors of ...

462

$$\begin{aligned}
 462 &= 2 \times 231 \\
 &= 2 \times 3 \times 77 \\
 &= 2 \times 3 \times 7 \times 11
 \end{aligned}$$

180

$$\begin{aligned}
 180 &= 2 \times 90 \\
 180 &= 2 \times 2 \times 45 \\
 180 &= 2 \times 2 \times 3 \times 15 \\
 180 &= 2 \times 2 \times 3 \times 3 \times 5
 \end{aligned}$$

If no common prime factors exist, $\text{GCD} = 1$ and the numbers are said to be *relatively prime*. Since 27 and 32 have no common prime factors, they are *relatively prime*.