Name_____ Class_____ Date_____

5-1

COMMON CORE

CC.7.G.4

Circumference of a Circle

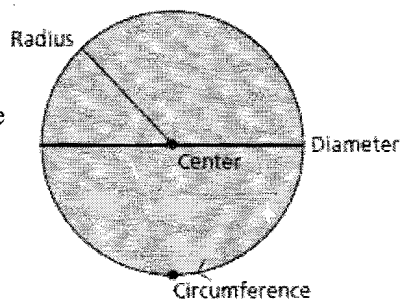
Essential question: *How do you find the circumference of a circle?*

Remember that a circle is a set of points in a plane that are a fixed distance from the center.

The diameter of a circle is a line segment that passes through the center of the circle and whose endpoints lie on the circle.

The radius is a line segment with one endpoint at the center of the circle and the other endpoint on the circle.

The circumference of a circle is the distance around the circle.



1 EXPLORE Exploring Circumference

A Use a measuring tape to find the circumference of five circular objects. Then measure the distance across each item to find its diameter. Record the measurements of each object in the table below. **Check students' work.**

Object	Circumference C	Diameter d	$\frac{C}{d}$

B Divide the circumference of each object by its diameter. Round your answer to the nearest hundredth.

C Describe what you notice about the ratio $\frac{C}{d}$ in your table.

Sample answer: $\frac{C}{d}$ is always close to or a little more than three.

REFLECT

1a. Conjecture Compare your results with other students. Make a conjecture about the relationship between the circumference and the diameter of a circle.

$\frac{C}{d}$ is always a little more than three.

1b. How could you estimate the circumference of a circular object without measuring it if you know the diameter?

Sample answer: Multiply the diameter by 3.

The ratio of the circumference to the diameter $\frac{C}{d}$ of any circle is the same for all circles. The ratio is called *pi*, or π. As you calculated in 1, the value of π is close to 3. You can approximate π as 3.14 or $\frac{22}{7}$. You can use this ratio to find a formula for circumference.

For any circle, $\frac{C}{d} = \pi$. Solve the equation for C to give an equation for the circumference of a circle in terms of the diameter.

$\frac{C}{d} = \pi$	The ratio of Circumference to Diameter is *pi*.
$\frac{C}{d} \times d = \pi \times d$	Multiply both sides by d.
$C =$ πd	Simplify.

Since the diameter is the same as two times the radius, you can also substitute $2r$ in the equation for d.

$d = 2r$	The diameter is two times the radius.
$C = \pi (2r)$	Substitute for d.
$C = 2\pi r$	Use the Commutative Property.

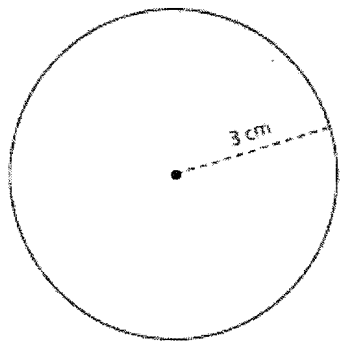
The two equivalent formulas for circumference are:

$C = \pi d$ and $C = 2\pi r$

2 EXAMPLE Finding the Circumference of a Circle

Find the circumference of the circle to the nearest hundredth. Use 3.14 for π.

The radius of the circle is 3 cm. Use the formula that includes the radius, $C = 2\pi r$.



$C = 2\pi r$	*Use the formula.*
$C = 2\pi 3$	*Substitute 3 for r.*
$C \approx 2(3.14)(3)$	*Substitute 3.14 for π.*
$C \approx 18.84$	*Multiply.*

The circumference is about 18.84 cm.

REFLECT

2a. What value of *pi* could you use to estimate the circumference? 3

2b. How do you know your answer is reasonable?

Sample answer: Using 3 for *pi*, $2 \times 3 \times 3 = 18$, which is close to the answer.

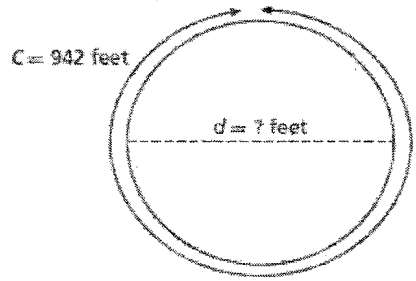
2c. When would it be logical to use $\frac{22}{7}$ instead of 3.14 for *pi*?

It would be logical to use $\frac{22}{7}$ when the radius or diameter is a multiple of 7.

3 EXPLORE Using Circumference

The circumference of a circular pond is 942 feet. A model boat is moving directly across the pond, along the diameter, at a rate of 4 feet per second. How long does it take the boat to get to the other side?

A Make a diagram.
Sketch the pond, and label what you know and what you need to find.



B First, you need to find the diameter of the pond. Use the formula for circumference, and solve for d.

$C = \pi d$	*Write the formula..*
$942 \approx 3.14\ d$	*Substitute for the circumference and pi..*
$\frac{942}{3.14} \approx \frac{3.14d}{3.14}$	*Divide both sides by* 3.14.
$300 \approx d$	*Simplify.*

The diameter is about 300 feet.

C Find the time it takes the boat to get across the pond traveling along the diameter. Divide the length of the diameter by the boat's speed.

$300 \div 4 = 75$

It takes the boat 75 seconds to get across.

REFLECT

3a. What If? How long would it take the boat to get across the pond if the model boat traveled at a rate of 5 feet per second?

It would take the boat 60 seconds to get across.

TRY THIS!

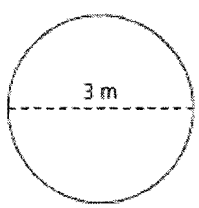
3b. The circumference of a circular garden is 42 meters. A gardener is using a machine to dig a straight line along the diameter of the garden at a rate of 10 meters per hour. How many hours will it take the gardener to dig across the garden?

about 1.34 hours

PRACTICE

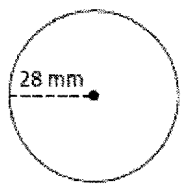
Find the circumference of each circle to the nearest tenth, if necessary. Use 3.14 or $\frac{22}{7}$ for π.

1.



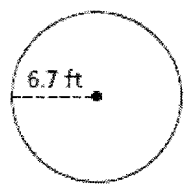
9.4 m

2.



176 mm

3.



42.1 ft.

4. In 1–4, which problems did you use $\frac{22}{7}$ for π? Explain.

Sample answer: I used $\frac{22}{7}$ in problem 2 since the radius was a multiple of 7.

Find each missing measurement to the nearest hundredth. Use 3.14 for π.

5. $r = 7$ m; $d =$ 14 m; $C \approx$ 43.96 m.

6. $r =$ 12.55 ft; $d =$ 25.10 ft; $C \approx 78.8$ ft

7. $r =$ 1.7 in; $d = 3.4$ in; $C \approx$ 10.68 in

8. $r = \frac{1}{2}$; $d =$ 1; $C \approx \pi$

9. A round fountain has a circumference of 6.28 meters. What is the diameter of the circle? Use 3.14 for π.

2 meters

10. Bob walks a circular path with a radius of 280 yards. Use $\frac{22}{7}$ for π.

a. How far does he walk? 1,760 yards

b. If he walks at a rate of 4 miles per hour, how long, in hours, will it take him to walk the circular path? *Hint:* 1,760 yards = 1 mile

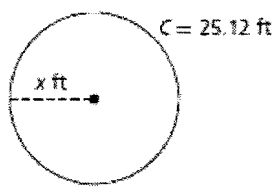
It will take him $\frac{1}{4}$ hour to walk 1 mile.

11. Carol wants to put ribbon around the top and bottom of a circular lampshade. The diameter of the shade is 21 inches. Use 3.14 for π.

a. Carol can only buy the ribbon in a whole number of inches. How much total ribbon must she buy? 132 in.

b. How much will she have to cut off? 0.12 in.

12. Error Analysis Kate says the radius of the circle is 8 feet. What is Kate's error? Find the correct radius of the circle.



Kate found the diameter of the circle, she needs to divide it by 2 to get the radius; 4 ft

Name_____ Class_____ Date_____

5-2
COMMON CORE

Area of a Circle

Essential question: *How do you find the area of a circle?*

CC.7.G.4

1 EXPLORE Finding the Area of a Circle

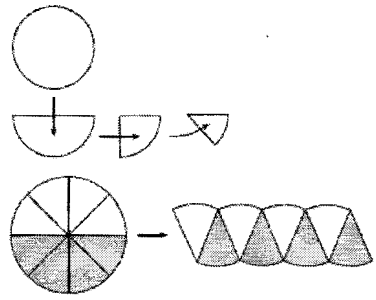
You can use what you know about circles and pi to help find the formula for the area of a circle.

Step 1: Use a compass to draw a circle and cut it out.

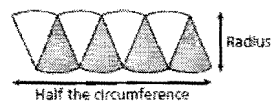
Step 2: Fold the circle three times as shown to get equal wedges.

Step 3: Unfold and shade one-half of the circle.

Step 4: Cut out the wedges and fit the pieces together to form a figure that looks like a parallelogram.



The base and height of the parallelogram relate to the parts of the circle.



base $b = \frac{1}{2}$ the circumference of the circle, or πr

height h = the radius of the circle, or r

To find the area of a parallelogram, the equation is $A = bh$.

To find the area of the circle, substitute for b and h in the area formula.

$A = bh$	
$A = \pi r \cdot h$	*Substitute πr for b.*
$A = \pi r \cdot r$	*Substitute r for h.*
$A = \pi r^2$	$r \cdot r = r^2$

REFLECT

1a. Conjecture Make a conjecture about the lengths of all the radii of a circle.

They all have the same measure.

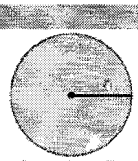
1b. How can you make the wedges look more like a parallelogram?

Sample answer: Make the wedges smaller so the base looks more like a straight line than curves.

Area of a Circle

The area of a circle is equal to π times the radius squared.

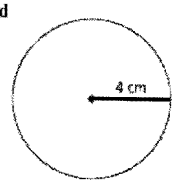
$A = \pi r^2$



Remember that area is given in square units.

2 EXAMPLE Finding the Area of a Circle

A biscuit recipe calls for the dough to be rolled out and circles to be cut from the dough. The biscuit cutter used is shown. Find the area of the biscuit once it is cut. Use 3.14 for π.



$A = \pi r^2$	*Use the formula.*
$A = \pi 4^2$	*Substitute. Use 4 for r.*
$A \approx 3.14 \times 4^2$	*Substitute. Use 3.14 for π.*
$A \approx 3.14 \times 16$	*Evaluate the power.*
$A \approx 50.24$	*Multiply.*

The area of the biscuit is about 50.24 cm^2.

TRY THIS!

2a. A flower garden is in the shape of a circle with a diameter of 10 yards. What is the area of the garden? Use 3.14 for π.

The area of the garden is about 78.5 yd^2.

2b. A circular pool has a radius of 10 feet. What is the area of the pool? Use 3.14 for π.

The area of the pool is about 314 ft^2.

REFLECT

2c. Compare finding the area of a circle when given the radius with finding the area when given the diameter.

The formula for area of a circle uses the radius of the circle. If you are given the diameter of the circle you must first divide it by two to get the radius.

2d. How you could estimate or check the reasonableness of an answer for the area of a circle?

Sample answer: You could substitute 3 for *pi*.

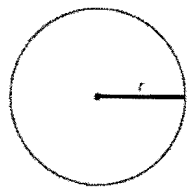
2e. Why do you evaluate the power in the equation before multiplying?

Sample answer: You must follow the order of operations and evaluate the exponents before multiplying.

You can use what you know about circumference and area of circles to find a relationship between them.

3 EXPLORE Finding the Relationship between Circumference and Area

Find the relationship between the circumference and area of a circle.



Start with a circle that has radius r.

Solve the equation $C = 2\pi r$ for r. $r = \frac{C}{2\pi}$

Substitute your expression for r in the formula for area of a circle. $A = \pi\left(\frac{C}{2\pi}\right)^2$

Square the term in the parenthesis. $A = \pi\left(\frac{C^2}{2^2 \cdot \pi^2}\right)$

Evaluate the power. $A = \frac{\pi \cdot C^2}{4 \cdot \pi^2}$

Simplify. $A = \frac{C^2}{4 \cdot \pi}$

Solve for C^2. $C^2 = 4\pi A$

The circumference of the circle squared is equal to four times pi times the area.

REFLECT

3a. Does this formula work for a circle with a radius of 3 inches? Show your work below.

Yes: $A = \pi r^2$ $C = 2\pi r$

$= \pi \cdot 3^2$ $= 2\pi 3$ $C^2 \stackrel{?}{=} 4\pi A$

$= 9\pi$ $= 6\pi$ $18.84^2 \stackrel{?}{=} 4 \times 3.14 \times 28.26$

≈ 28.26 ≈ 18.84 $354.9456 = 354.9456$

TRY THIS!

Find the area of the circles given the circumference. Give your answers in terms of π.

3b. $C = 8\pi$; $A =$ 16π square units

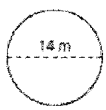
3c. $C = \pi$; $A =$ $\frac{\pi}{4}$ square units

3d. $C = 2\pi$; $A =$ π square units

PRACTICE

Find the area of each circle to the nearest tenth, if necessary. Use 3.14 for π.

1.

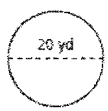


153.9 m^2

2.

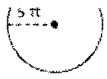
452.2 mm^2

3.



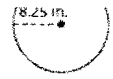
314 yd^2

4. **5.** **6.**



78.5 ft^2

322 cm^2



213.7 in^2

7. A clock face has a radius of 8 inches. What is the area of the clock face? Round your answer to the nearest hundredth.

200.96 in^2

8. A DVD has a diameter of 12 centimeters. What is the area of the DVD? Round your answer to the nearest hundredth.

113.04 cm^2

9. A company makes steel lids that have a diameter of 13 inches. What is the area of each lid? Round your answer to the nearest hundredth.

132.67 in^2

10. A circular garden has an area of 64π square yards. What is the circumference of the garden? Give your answer in terms of π.

16π yd

11. **Reasoning** A small silver dollar pancake served at a restuarant has a circumference of 2π inches. A regular pancake has a circumference of 4π inches. Is the area of the regular pancake twice the area of the silver dollar pancake? Explain.

No, the area of the regular pancake is 4 times as large as the area of the silver dollar pancake.

12. **Critical Thinking** Describe another way to find the area of a circle when given the circumference.

Sample answer: First find the radius of the circle by using the formula $C = 2\pi r$. Then substitute the radius into the formula for the area of a circle.

Name_____ Class_____ Date_____

5-4

Solving Surface Area Problems

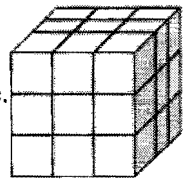
COMMON CORE

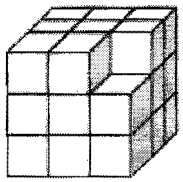
Essential question: *How do you find the surface area of a figure made of prisms?*

CC.7.G.6

1 EXPLORE Comparing the Surface Area of Two Figures

Using centimeter cubes, build the two figures shown.





A Find the surface area of the 3-by-3-by-3 cube.

54 cm^2

B Now find the surface area of the cube with one missing corner.

54 cm^2

C Which figure has a greater surface area: the 3-by-3-by-3 cube, or the same cube with one of the corners missing?

They are the same.

REFLECT

1a. How did you find the surface area of the figures?

Sample answer: I counted the faces of the centimeter cubes that were showing.

1b. Why does it make sense that the surface areas are equal?

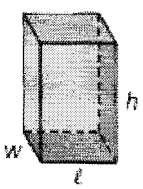
Sample answer: The number of centimeter-cube faces that show is the same for both figures.

1c. What If? If four cubes are taken, one from each corner of the top layer, would this change the surface area?

Sample answer: No, the number of centimeter-cube faces that show would still be the same.

One way to find the surface area of a figure is to make a net, open it up, find the areas of the shapes, and add them together. Another way to find the surface area is to use a formula.

Consider a rectangular prism with length, l, width, w, and height, h. The top and bottom faces have the same area, $A = l \ w$. The front and back faces have the same area, $A = l \ h$.
The left and right faces have the same area, $A = w \ h$.

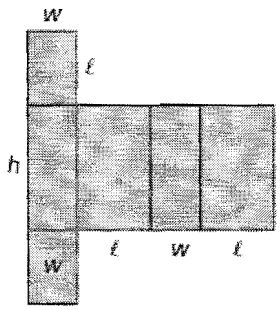


To find the surface area, add the areas of the top, bottom, front, back,

left, and right faces.

$S = lw + lw + lh + lh + wh + wh$

top bottom front back left right



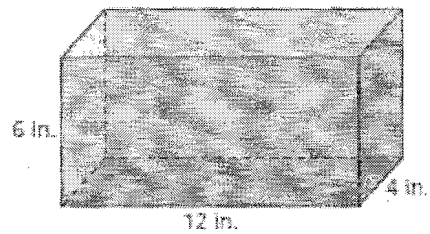
Combine like terms to find the formula for surface area of a rectangular prism.

$S = 2lw + 2lh + 2wh$

2 EXAMPLE Finding the Surface Area of a Rectangular Prism

Felix is making a jewelry box out of balsa wood as a present for his sister. He wants the jewelry box to be 12 inches long, 4 inches wide, and 6 inches tall. How much balsa wood does Felix need?

Step 1: Sketch and label the prism.



Step 2: Find how much balsa wood Felix needs to make his box.

- Use the formula for surface area of a rectangular prism.

  $S = 2lw + 2lh + 2wh$

- Substitute for the length, width, and height.

  $S = 2(12 \cdot 4) + 2(12 \cdot 6) + 2(4 \cdot 6)$

- Simplify each term.

  $S = 96 + 144 + 48$

- Add.

  $S = 288$

Felix needs 288 in^2 of balsa wood for his jewelry box.

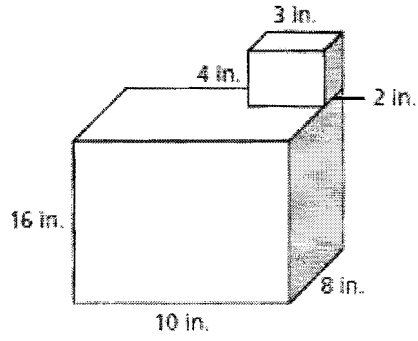
REFLECT

2. Adapt the formula for the surface area of a rectangular prism for a cube. What is the formula for the surface area of a cube?

$S = 6lw$

3 EXAMPLE Finding the Surface Area of a Composite Solid

Matthew builds a model of a simple flat-roofed house with a chimney on top. He wants to paint both the house and chimney with red paint. How many square inches will he paint?



A Find the surface area of the chimney.

$S = 2lw + 2lh + 2wh$

$S = 2(3 \cdot 2) + 2(3 \cdot 4) + 2(2 \cdot 4)$

$S = 12 + 24 + 16$

$S = 52$

The surface area of the chimney is 52 square inches.

B Find the surface area of the house. Do not include the bottom of the house.

$S = lw + 2lh + 2wh$

$S = (10 \cdot 8) + 2(10 \cdot 16) + 2(8 \cdot 16)$

$S = 80 + 320 + 256$

$S = 656$

The surface area of the house is 656 square inches.

C Add the surface areas of the chimney and the house.

$S = 52 + 656 = 708$

D Part of the chimney and house overlap. The overlapping area has a length of 3 inches and a width of 2 inches, or an area of 6 square inches. Subtract two times that area.

$S = 708 - 2(6) = 696$

Matthew will paint 696 square inches.

REFLECT

3a. Explain why you subtract the overlap area two times.

Sample answer: You have to subtract it from the surface area of both the chimney and the house, or two times.

TRY THIS!

3b. Matthew decides to add an extension to the right side of the house that is 12 inches tall, 6 inches long, and 4 inches wide. If he repaints the model blue, not including the bottom, how many square inches will

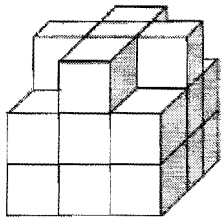
he paint?

864 square inches

PRACTICE

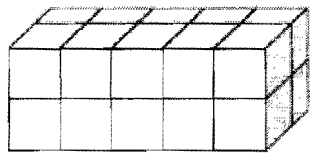
Find the surface area of each figure.

1.



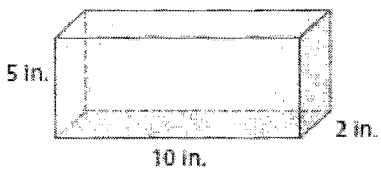
54 cm^2

2.



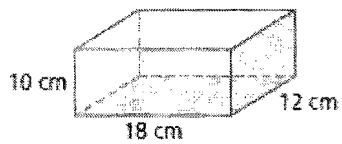
48 cm^2

3. Carla is wrapping a present in the box shown below. Find the amount of wrapping paper she needs, not counting overlap.



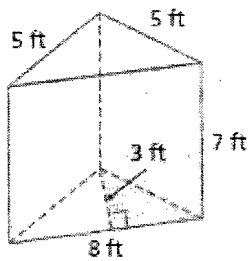
160 in^2

4. Henry plans to cover the box shown below in contact paper without any overlap. How many square centimeters will be covered with contact paper?



1,032 cm^2

5. To find the surface area of a triangular prism use the formula $S = 2B + Ph$. B is the area of the base, P is the perimeter of the bases, and h is the height of the prism.



a. The height of the prism is 7 ft.

b. The area of the base is 12 ft^2.

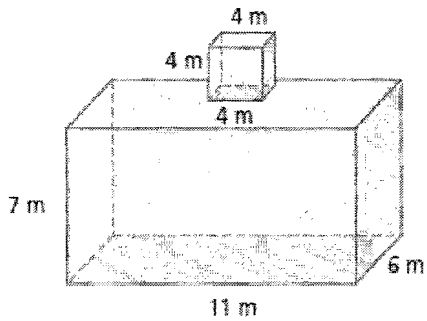
c. The perimeter of the base is 18 ft.

d. Fill in the formula. $S = 2 \cdot$ 12 + 18 7

e. The surface area of the triangular prism is 150 ft^2.

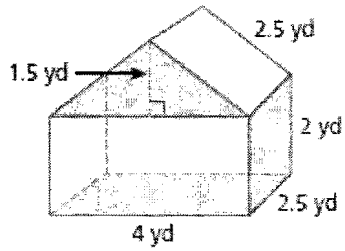
Find the surface area of each composite figure.

6.



434 m^2

7.



54.5 yd^2

Name_____ Class_____ Date_____

5-5
COMMON CORE

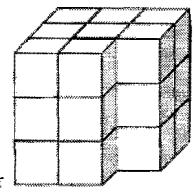
Solving Volume Problems

Essential question: *How do you find the volume of a figure made up of cubes and prisms?*

CC.7.G.6

1 EXPLORE Finding the Volume of a Prism

A Use centimeter cubes to build a prism like the one shown. Each cube represents a unit of measure called a cubic unit, so centimeter cubes represent cubic centimeters.



Step 1: Find the volume of the prism. Count the number of cubes that make up the prism.

The volume of the prism is 24 cm^3.

Step 2: Find the area of the base by counting the number of cubes that make up the face of the top or bottom of the prism.

The area of the base is 8 cm^2.

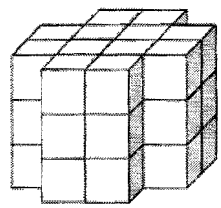
Step 3: Find the height of the prism.

The height of the prism is 3 cm.

Do you see a relationship between the volume and the area of the base and the height of the prism?

The volume is equal to the area of the base times the height.

B Following the steps in A, find the volume, area of the base, and height of the given prism.



Volume: 36 cm^3

Area of the base: 12 cm^2

Height of the prism: 3 cm

Do you see a relationship between the volume and the area of the base and the height of the prism?

The volume is equal to the area of the base times the height.

REFLECT

1a. Conjecture Based on your discoveries in 1, describe in words a way to find the volume of any prism.

Volume is the area of the base times the height.

You can find the volume of any prism by multiplying the area of the base B by the height of the prism *h*.

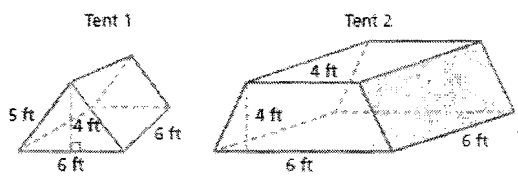
Volume of a Prism

The volume *V* of a prism is the area of its base B times its height *h*.

$$V = Bh$$

2 EXAMPLE Finding the Volume of Prisms

Bradley is setting up two tents. One is the shape of a triangular prism and the other is the shape of a trapezoidal prism. How many cubic feet of space are in each tent?



A Find the volume of Tent 1.

$V = Bh$ *Use the formula.*

$V = \left(\frac{1}{2}bh\right)h$ *The base is a triangle.*

$V = (\frac{1}{2}\ 6\ \ 4\ )h$ *Substitute for b and h in the base.*

$V = (\ 12\ )(\ 6\ )$ *Substitute for the height of the prism, h.*

$V = 72$ *Multiply.*

The volume of Tent 1 is 72 ft^3.

B Find the volume of Tent 2.

$V = B\ h$	*Use the formula.*
$V = \left(\frac{1}{2}h(b_1 + b_2)\right)h$	*The base is a trapezoid.*
$V = \left(\frac{1}{2}\ 4\ (6 + 4)\right)h$	*Substitute for h, b_1, and b_2 in the base.*
$V = (20)(6)$	*Substitute for the height of the prism, h.*
$V = 120$	*Multiply.*

The volume of Tent 2 is 120 ft^3.

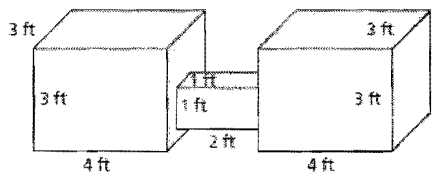
REFLECT

2. For a prism that is not a rectangular prism, how do you determine which sides are the bases? For a rectangular prism, how do you determine which sides are the bases?

Sample answer: The non-rectangular sides are the bases. For a rectangular prism, any pair of opposite sides can be considered the bases.

3 EXAMPLE Finding the Volume of a Composite Solid

Allie has two aquariums connected by a small square prism. Find the volume of the double aquarium.



A Find the volume of each of the larger aquariums.

$V = Bh$	*Use the formula.*
$V = (12)(3)$	*Substitute for B and h.*
$V = 36$	*Multiply.*

The volume of each end aquarium is 36 cubic feet.

B Find the volume of the connecting prism.

$V = Bh$	*Use the formula.*
$V = (1)(2)$	*Substitute for B and h.*
$V = 2$	*Multiply.*

The volume of the connecting prism is 2 cubic feet.

C Add the volume of each part of the aquarium.

$$V = 36 + 36 + 2 = 74$$

The volume of the aquarium is 74 cubic feet.

REFLECT

3a. What if? Find the volume of the aquarium if all of the dimensions were doubled. What is the relationship between the original volume and the new volume?

592 ft^3; the new volume is eight times the original volume.

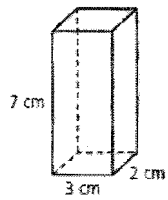
3b. Find the volume of one of the end aquariums using another pair of opposite sides as the base. Do you still get the same volume? Explain.

36 ft^3; yes, if you use a base that is 3 ft by 3 ft, the height of the prism is 4 ft. $V = (3 \times 3)(4) = 9 \times 4 = 36$

PRACTICE

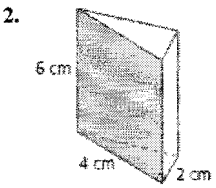
Find the volume of each figure.

1.



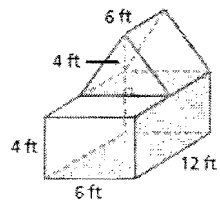
42 cm^3

2.



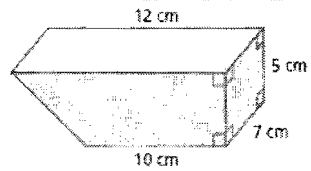
42 cm^3

3.



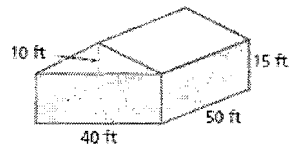
360 ft^3

4. Pete fills the container shown with sand. How much sand fills the container?



385 cm^3

5. Mr. Fowler is building a barn for his farm. The dimensions are shown at right. Find the volume of the entire barn.



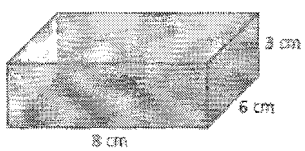
40,000 ft^3

6. A movie theater offers popcorn in two different containers for the same price. One container is a rectangular prism with a base area of 36 in^2 and a height of 5 in. The other container is a triangular prism with a base area of 32 in^2 and a height of 6 in. Which container is the better deal? Explain.

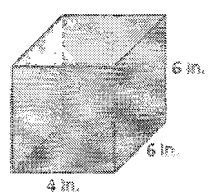
The triangular prism container is the better deal because the volume is 192 cubic inches, and the rectangular prism only holds 180 cubic inches.

7. Critical Thinking Can rectangular prisms have different heights and the same volume? Show your work below.

Yes, different rectangular prisms can have different heights and the same volume.
Sample answer:



$V = (8)(6)(3) = 144\ cm^3$



$V = (4)(6)(6) = 144\ cm^3$